

Angles of the legs and chest contours of striding figures in Old Kingdom reliefs

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1 Introduction

In ancient Egyptian two-dimensional images, the viewer always gets to see the essence of the figure, animal, or object being depicted ("aspective" view).² For human figures, this meant a characteristic twist: the whole figure was shown in side view, but the eye and the upper body were shown in frontal view. In the stride position (for male figures³), the legs form a certain angle. Both soles stand on the base line⁴, with the viewer looking at the big toe and the arch of each (!) foot.⁵

Unfinished reliefs and murals in tombs of all periods show that Egyptian artists⁶ used horizontal guide lines or a grid to sketch the figures on the wall.⁷ Remnants of these lines can sometimes be seen in finished works where the background paint has flaked off.⁸ After smoothing the background, the first step was to apply the guide lines in red. The next step was to sketch the figures of the scene, also in red, but often corrected in black (master and

² Cf. Binder 2000, 29–33; Robins 1994, 3–8.

³ The striding posture was probably intended to express activity. Women were almost always depicted with their legs together, but see e.g. Darlow 2017, 216 (fig. 5.20, right).

⁴ Since the Early Dynastic Period, a base line was obligatory. Cf. Darlow 2017, 177; Robins 1994, 6. Only in the art of the Intermediate Periods could it occasionally be absent. Cf. Darlow 2017, 178.

⁵ Until the 18th dynasty, no importance was attached to the natural side differences of hands and feet. People were satisfied with the "idea" of a hand or foot. Cf. Kanawati /Woods 2009, 30; Robins 1994, 13–15.

⁶ In ancient Egypt, those who made the wall decorations were gifted painters and craftsmen who (except during the Intermediate Periods) had to adhere to certain specifications. Nevertheless, I call them "artists" in admiration of their work.

⁷ In Old Kingdom examples, one finds a few mostly horizontal guide lines, very rarely a complete grid, but mostly none. Cf. Baud 1935, 44–58; Blackman 1915, Plates 10, 11, 15; Darlow 2017, 183–185; Robins 1994, 60, 64–69 and figures 4.1–4.5. However, sometimes a grid is younger than the tomb, because the artists copied older scenes. Cf. Kanawati 2011, 493–495. The oldest known remnant of a grid, certainly from the Old Kingdom, is found in the tomb of Pepyankh-Henykem (Meir A2) in a scene of the tomb owner spearfishing. Cf. Darlow 2017, 201–202. The grids in the rock tombs at Meir date from the 12th dynasty. Cf. Darlow 2017, 182; Robins 1994, 64, 70.

⁸ Cf. Robins 1994, 30.

apprentice⁹). After the areas were filled with color or the relief was hammered, the contours of the figures were again finely traced with black color.¹⁰

1.1 The canon of proportions in the Old Kingdom

In the 19th century, the French engineer Achille Émile Prisse d'Avennes and the art expert Charles Blanc were among the first to publish on the proportions of figures in ancient Egyptian wall decorations. They challenged the prevailing view that a grid was merely a means of transferring a figure sketched on a small scale to the large scale of the wall.¹¹ According to them, the height of human figures from the soles to the crown of the head was divided into 19 parts. Prisse d'Avennes wrote that the knees were set at $\frac{6}{19}$, the genitals at $\frac{9.5}{19}$, and the shoulders at $\frac{16}{19}$.¹² Blanc assigned a specific anatomical detail to each horizontal line of the grid, so his scheme differed somewhat from that of Prisse d'Avennes.¹³ The German Egyptologist Karl Richard Lepsius (1884) recognized the importance of the frontal hairline and divided its height into 6 parts. He postulated that the length of a foot was $\frac{1}{6}$ of this height and served as a module for the entire composition.¹⁴ In contrast, the Scotsman Campbell Cowan Edgar (1905) concluded that the figures may have followed a set of construction rules, but that no one part of the body served as a reference measure for all others.¹⁵ It soon became clear that the canon of proportions had not remained the same throughout the ages of ancient Egypt.¹⁶ In 1955, Erik Iversen stated that in Old Kingdom figures the grid spacing is equal to the transverse diameter of a vertically held fist, and the length of a foot and the length of a forearm from elbow to wrist is 3 squares. Three squares are 4 palm widths and 16 finger widths. Furthermore, the male forearm from the elbow to the tip of the thumb is $4\frac{1}{2}$ squares (24 finger widths), and this, Iversen wrote, is the equivalent to the ancient Egyptian (non-royal) cubit of about 45 cm.¹⁷

The work of Gay Robins (1994) confirmed what Karl Richard Lepsius had already

⁹ Cf. Pieke 2011, 222-225.

¹⁰ Cf. Robins 1994, 26. In the case of reliefs on exterior walls, any paint has usually disappeared completely.

¹¹ Cf. Robins 1994, 31, 37-39.

¹² Cf. Robins 1994, 31-32, quoted from Prisse d'Avennes 1879, 122-129.

¹³ Cf. Blanc 1876, 43.

¹⁴ Cf. Robins, 31, 35-36 and fig. 2.5, quoted from Lepsius 1884, Appendix.

¹⁵ Cf. Robins 1994, 38, quoted from Edgar 1905.

¹⁶ Cf. Robins 1994, 34-36.

¹⁷ Cf. Robins 1994, 41-42, quoted from Iversen 1955, 22-27. Applied to the living, this would mean that the average height of the frontal hairline was more than 180 cm (18 man's fists or 6 feet). This cannot be confirmed by the measurements of ancient Egyptian skeletons. Cf. Raxter 2011, 124 (table 10). Robins criticized Iversen's hypotheses for additional reasons. Cf. Robins 1994, 45-56.

recognized in the basics in 1884. The following is now undisputed for the art of the Old and Middle Kingdoms:

- The height of the frontal hairline (h) was used as the relevant body height.¹⁸
- The vertical coordinates of the major body parts of a standing or striding Old and Middle Kingdom figure are based on the unit $\frac{1}{18} h$.
- The knees are set at $\frac{6}{18} h$, the gluteal fold at $\frac{9}{18} h$, the waist at $\frac{11}{18} h$, the elbow of a hanging arm at $\frac{12}{18} h$, the nipple¹⁹ at $\frac{14}{18} h$, and the base of the neck at $\frac{16}{18} h$ (Fig. 1).²⁰
- In general, these positions apply to female figures as well.²¹

In this article, the construction horizontals are named after the numerators of these fractions: H0, H1, H2, H3, ... H18 (Fig. 1).

There was no strict order for the coordinates in the horizontal plane: the positions of the heels, waist, nipple, and shoulders were not uniformly placed on certain perpendiculars of a grid based on the unit $\frac{1}{18} h$, unless the grid spacings were further subdivided.²² The purpose of this study was to find simple design principles for the positioning of the heels, the waist, the nipple and the armpits, as well as the inclination of the anterior and posterior chest contours.²³

¹⁸ The height measured to the highest point of the head was not practical due to the different headgear and crowns.

¹⁹ Although the chest is shown in frontal view, the breast remains in side view. Thus, only one nipple is visible.

²⁰ Cf. Darlow 2017, 188–190, 198; Robins 1994, 64, 73–74, 76 and fig. 4.8.

²¹ Cf. Robins 1994, 75 (fig. 4.10), 79, 81 (fig. 4.15). In female figures, the gluteal fold is sometimes at $\frac{10}{18} h$ with the other proportions unchanged. Cf. Robins 1994, 79 (fig. 4.13).

²² Unlike Iversen, Robins did not define a vertical midline. He pointed out that the adjacency of body parts to grid lines could be random, or that a slight shift in the grid could cause this (consciously or unconsciously). Cf. Robins 1994, 62.

²³ Strictly speaking, the area between the waist and the armpits is a transition from the side view to the front view.

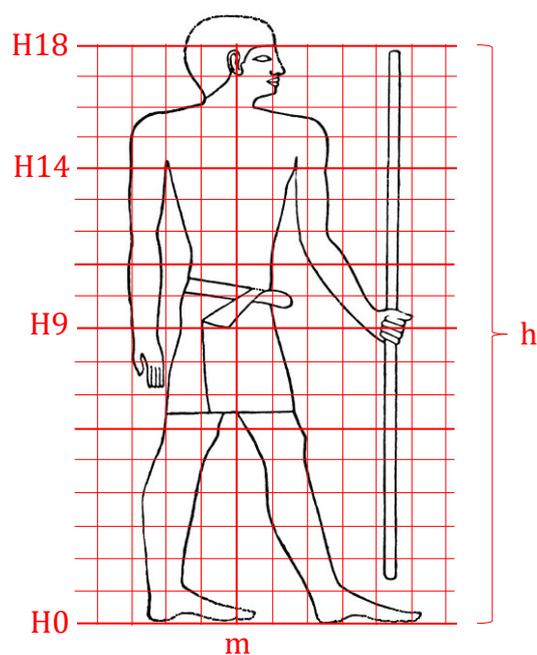


Fig. 1: Striding figure within a grid based on 1/18 of the height of the frontal hairline.

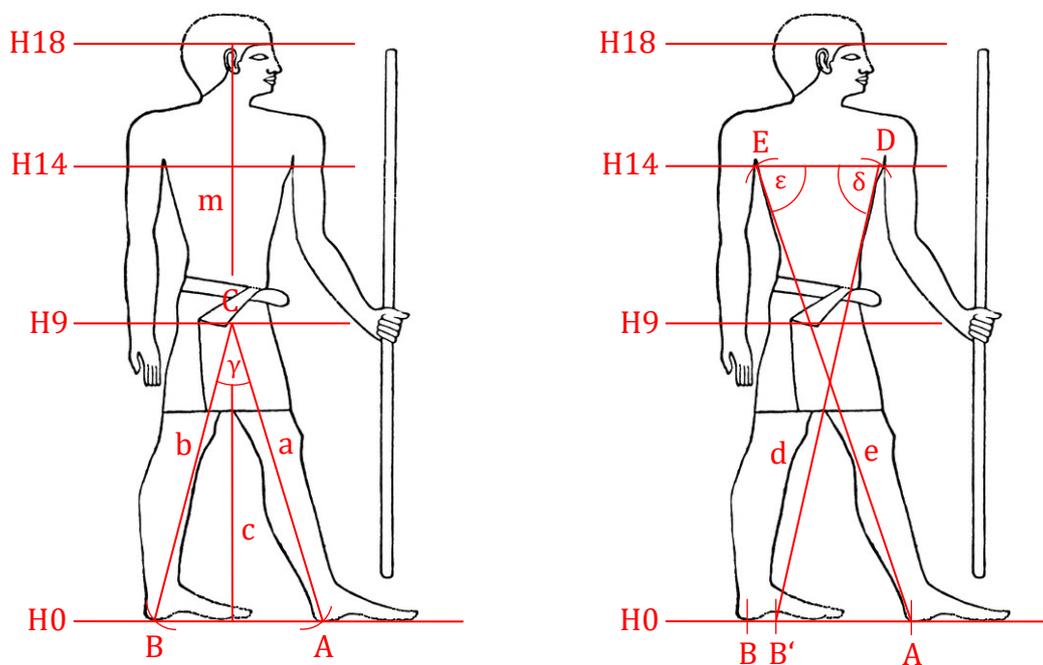


Fig. 2: Hypothetical construction lines of striding figures.

These are the basic observations made on Old Kingdom striding figures (Fig. 2):

A figure has a vertical midline (m), which is a perpendicular through the indentation between the ball of the big toe and the ball of the rear forefoot. The distance between the base line (H0) and a horizontal through the frontal hairline (H18) is the height h, the 18th part of which is the grid spacing (module). The horizontal lines H0, H9 and H14 define the vertical coordinates of points A to E. Points A, B, and C are the corners of an isosceles triangle, with its apex (C) on the vertical midline. The angle γ (stride angle) between the two legs is 25–30°. Points D and E are defined as the intersection of the construction lines d and e with H14.²⁴ These construction lines also serve as guides for the inclination of the posterior and anterior chest contours. The angles that e and d form with H14 (δ , ϵ) are 70–80°.²⁵

1.2 Hypotheses

The following hypotheses were derived from the preliminary examination of five Old Kingdom striding figures:

Hypothesis 1: The angles γ , δ and ϵ were canonical.

Hypothesis 2: To obtain the canonical angle γ , points A and B on the base line (H0) were marked with a compass. The radius r of an arc centered on C has a fixed length ratio to c, with $r = a = b = c + u$ (Fig. 3).²⁶ The distances c, u, and the radius r ($= a = b = c + u$) have the following mathematical relationship (angle γ in radians²⁷):

$$u = r \left(1 - \cos\left(\frac{\gamma}{2}\right) \right)$$

$$\frac{u}{c + u} = 1 - \cos\left(\frac{\gamma}{2}\right)$$

²⁴ The horizontal coordinate of E is only occasionally exactly above point B, but usually well in front of or behind point B.

²⁵ See

Tab. 1 in the methods section (Chapter 2) for the definition of each point and line.

²⁶ In Figure 3 of this article, the distance u is drawn larger than it actually is for better visualization.

²⁷ Conversion of angular dimensions: Angle (rad) \approx Angle (°) $\times \frac{\pi}{180}$; Angle (°) \approx Angle (rad) $\times \frac{180}{\pi}$ (spreadsheet programs have built-in conversion functions).

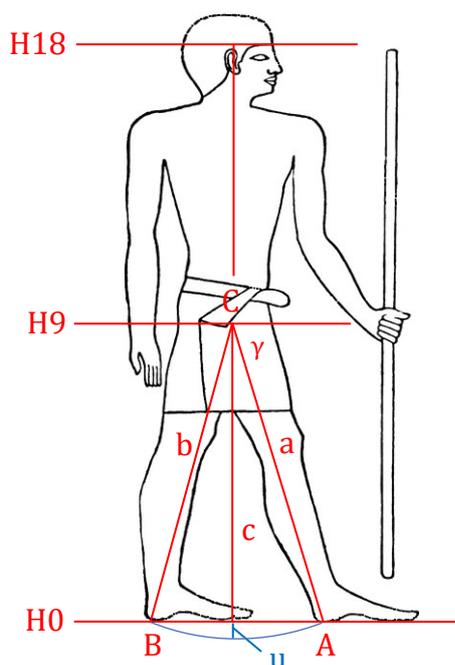


Fig. 3: Hypothetical construction of the heel points A and B.

Hypothesis 3: Line d is the radius of an arc centered on B' used to mark point D on $H14$. Line e is the radius of an arc centered on A used to mark the point E on $H14$. The lengths of d and e are determined by the fixed (canonical) length ratios d/b and e/a . This also makes the angles δ and ϵ canonical. Lines d and e also serve as guides for the inclination of the posterior and anterior chest contours. See Fig. 2.

Hypothesis 4: The shoulder width (f) and the waist width (g) are not directly related to $\frac{1}{18} h$, but were drawn freehand with observation of the adjacent horizontals and points (Fig. 4).

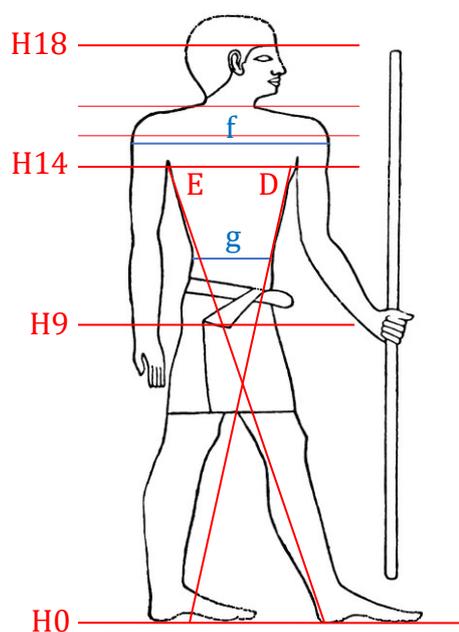


Fig. 4: Shoulder and waist widths.

2 Method

In order to substantiate the hypotheses, photos of a total of 24 striding figures with hanging or slightly forward pointing upper arms from the 3rd to 6th dynasties were selected. In addition to figures on tomb walls, two stelae, a pillar and a loose block fragment were included to demonstrate the universal validity of the construction rules. The photos were taken from internet sources (photobooks available online, museum websites)²⁸ and have been converted to JPEG format where necessary. Measurements were made using the image processing program GIMP (version 2.10.34; <https://www.gimp.org/>).²⁹ Table 1 provides an overview of the definition of the landmarks and variables determined on the figures. For descriptive statistical analysis, means, standard deviations (SD), relative standard deviations (RSD), medians, and ranges were calculated.

²⁸ The photos are not shown here due to copyright issues.

²⁹ Only length ratios and angles were relevant to the question. Therefore, the absolute size of a photo and the absolute lengths of construction lines did not play a role in the evaluation.

Tab. 1: Landmarks and measured variables.

Element	Type	Name	Definition
H0	Horizontal	Base line	Base line of the figure
H9	Horizontal	Gluteal level	Horizontal at the level of the gluteal fold
H14	Horizontal	Chest horizontal	Horizontal at nipple height
H18	Horizontal	Hairline horizontal	Horizontal at the level of the frontal hairline
m	Vertical	Vertical midline	Vertical through the indentation between the ball of the big toe and the ball of the rear forefoot
h	Vertical	Height of the frontal hairline	Distance H0→H18
A	Point	Front heel point	Point of contact of the front heel with the base line ³⁰
B	Point	Rear heel point	Point of contact of the rear heel with the base line
B'	Point	Rear arch point	Intersection of a perpendicular through the highest point of the rear foot arch with H0
C	Point	Central point	Intersection of m with H9
D	Point	Nipple point	Intersection of d with H14
E	Point	–	Intersection of e with H14
a	Straight line	Anterior leg of the leg triangle	Distance A→C
b	Straight line	Posterior leg of the leg triangle	Distance B→C
c	Straight line	Height of the leg triangle	Distance H0→C (H0→H9)
d	Straight line	Construction line d	Distance B'→D
e	Straight line	Construction line e	Distance A→E
f	Straight line	Shoulder width	Largest horizontal distance between the outer shoulder contours
g	Straight line	Waist width	Smallest horizontal distance between the lumbar lordosis and the abdominal wall
γ	Angle	Stride angle	Angle between a and b
δ	Angle	Front chest angle	Angle between d and H14
ε	Angle	Rear chest angle	Angle between e and H14

³⁰ If the heel is round, a contact point can be defined. Otherwise, the center of the area where the heel touches the baseline is used.

3 Results

In the 24 Old Kingdom figures examined, the height of H9 (gluteal fold) is 0.51 ± 0.02 h and the height of H14 (nipple) is 0.77 ± 0.01 h. The results relevant to the hypotheses are summarized in tables (Tab. 2, Tab. 3).

3.1 Results for hypotheses 1 to 3

If the proposed construction methods for points A, B, D and E are correct, the angles γ , δ and ε and the distance ratios d/b and e/a must be largely constant for all the striding figures studied. Table 2 shows the results for these parameters.

Tab. 2: Angles and distance ratios in Old Kingdom striding figures (n=24).

#	Source	Dyn.	γ (°)	δ (°)	ε (°)	d/b	e/a
1	Tomb of Nefer	4	30.0	77.4	71.4	1.53	1.58
2	Tomb of Nefer	4	27.1	77.8	73.5	1.51	1.54
3	Pillar of Pepi II	6	27.0	77.3	72.0	1.45	1.49
4	Pyramid temple of Sahure	5	28.1	77.5	71.8	1.48	1.53
5	Tomb of Ti	5	27.7	80.5	70.2	1.37	1.44
6	Tomb of Khufukhaf	4	31.5	78.1	70.8	1.51	1.56
7	Stela of Qahedjet (Huni)	3	28.8	80.1	73.1	1.63	1.69
8	Limestone building block ³¹	4	29.2	78.5	73.3	1.59	1.62
9	Tomb of Tepemankh	5	25.6	79.6	74.6	1.47	1.50
10	Tomb of Tepemankh	5	27.7	78.5	72.6	1.53	1.57
11	Tomb of Metjetji	5	28.2	78.7	73.2	1.52	1.55
12	Tomb of Metjetji	5	27.6	79.4	72.8	1.56	1.59
13	Tomb of Seshseshet Idut	6	24.7	78.9	72.9	1.47	1.51
14	Tomb of Seshseshet Idut	6	26.0	77.1	74.2	1.54	1.56
15	Tomb of Kagemni	6	28.0	77.3	73.3	1.46	1.50
16	Tomb of Mereruka	6	28.4	76.2	73.1	1.56	1.59
17	Tomb of Mereruka	6	29.8	76.4	72.5	1.54	1.57
18	Tomb of Ptahhotep	5	24.6	77.3	75.0	1.49	1.51
19	Tomb of Ptahhotep	5	30.2	80.1	69.0	1.52	1.60
20	Tomb of Mereruka	6	29.4	76.3	72.5	1.51	1.53

³¹ Possibly from the pyramid precinct of Snofru at Dahshur, reused by Amenemhet I at el-Lisht.

#	Source	Dyn.	γ (°)	δ (°)	ϵ (°)	d/b	e/a
21	Stela of Meni	6	26.9	77.8	73.6	1.55	1.57
22	Tomb of Meriteti	6	27.2	78.2	73.6	1.50	1.53
23	Tomb of Meriteti	6	29.0	78.5	72.2	1.52	1.57
24	Tomb of Nefer	4	31.1	77.6	71.3	1.51	1.56
		Mean	28.1	78.1	72.6	1.51	1.55
		SD	1.8	1.2	1.4	0.05	0.05
		Median	28.0	78.0	72.8	1.51	1.56
		Range	24.6, 31.5	76.2, 80.5	69.0, 75.0	1.37, 1.63	1.44, 1.69

Dyn.: Dynasty; SD: Standard deviation.

3.2 Results for hypothesis 4

If hypothesis 4 is correct, then the ratios of shoulder width (f) to h, or waist width (g) to h, or g to f are not constant. Table 3 presents these parameters.

Tab. 3: Ratios of shoulder and waist widths.

#	Source	Dyn.	f/h	g/h	g/f
1	Tomb of Nefer	4	0.34	0.13	0.37
2	Tomb of Nefer	4	0.32	0.13	0.40
3	Pillar of Pepi II	6	0.32	0.11	0.35
4	Pyramid temple of Sahure	5	0.34	0.12	0.35
5	Tomb of Ti	6	NA	0.12	NA
6	Tomb of Khufukhaf	4	0.35	0.13	0.38
7	Stela of Qahedjet (Huni)	3	0.31	0.13	0.39
8	Limestone building block	4	0.32	0.13	0.41
9	Tomb of Tepemankh	5	0.31	0.12	0.39
10	Tomb of Tepemankh	5	0.30	0.12	0.40
11	Tomb of Metjetji	5	0.32	0.12	0.37
12	Tomb of Metjetji	5	NA	0.11	NA
13	Tomb of Seshseshet Idut	6	0.33	0.13	0.40
14	Tomb of Seshseshet Idut	6	0.33	NA	NA
15	Tomb of Kagemni	6	0.34	0.13	0.39
16	Tomb of Mereruka	6	0.35	0.14	0.40

#	Source	Dyn.	f/h	g/h	g/f
17	Tomb of Mereruka	6	NA	0.15	NA
18	Tomb of Ptahhotep	5	0.35	0.13	0.36
19	Tomb of Ptahhotep	5	NA	0.13	NA
20	Tomb of Mereruka	5	0.33	0.13	0.40
21	Stela of Meni	6	0.33	0.13	0.40
22	Tomb of Meriteti	6	NA	0.13	NA
23	Tomb of Meriteti	6	0.34	0.13	0.37
24	Tomb of Nefer	4	0.32	0.12	0.39
		Mean	0.33	0.13	0.38
		SD	0.01	0.01	0.02
		Median	0.33	0.13	0.39
		Range	0.30, 0.35	0.11, 0.15	0.35, 0.41

Dyn.: Dynasty; NA: Not assessable; SD: Standard deviation.

4 Discussion

The present work deals with hypothetical construction methods of striding figures in Old Kingdom reliefs. The study confirms the long-known observation that in the Old Kingdom, the partial heights of a standing or striding figure were measured in multiples of $\frac{1}{18}$ h. The good agreement between the means and the medians indicates that there were no significant outliers. Therefore, it is justified to discuss only the means and SD. The RSD (SD/mean [%]) serves as an intuitive measure of precision.

The heights of the horizontals H9 and H14 meet their targets³² with relatively high accuracy: H9: target 0.5 h, measured value 0.51 ± 0.02 h, RSD 4.4%; H14: target 0.78 h, measured value 0.77 ± 0.01 h, RSD 1.7%. Thus, the average deviation from target is 2% and -1.3%, respectively.³³

4.1 Discussion of the hypotheses

As shown in Figure 1, the vertical midline (m) of a striding figure is defined as the

³² Cf. Robins 1994, 64, 73–74, 76 and fig. 4.8.

³³ The heights of H6 (knees) and H16 (base of the neck) in this study meet their targets (0.33 h, 0.89 h) but do not contribute to the research question. There are examples from Old Kingdom tombs where it can be seen with the naked eye that the proportions of the head and chest are incorrect in a number of offering bearers. Cf. Robins 1994, 68 (figs. 4.4, 4.5).

perpendicular on the base line through the indentation between the ball of the big toe and the ball of the rear forefoot.³⁴ Lepsius and Iversen drew the vertical midline in the same way.³⁵ In a more recent thesis, Darlow named the anterior edge of the ear as the reference.³⁶

The constancy of the angles γ ($28.1 \pm 1.8^\circ$, RSD 6.4%), δ ($78.1 \pm 1.2^\circ$, RSD 1.5%), and ϵ ($72.6 \pm 1.4^\circ$, RSD 1.9%) supports hypothesis 1. The precision of the angle γ in the absence of horizontal alignment of points A and B with a vertical grid line or anatomical detail is well consistent with hypothesis 2 that these points were marked on the base line with an arc with center C. The fact that the leg triangle is a highly accurate isosceles triangle ($a/b = 1.0 \pm 0.01$, RSD 0.9%) also supports this construction method. The length of the feet is given by the distance $m \rightarrow B$, to which only the big toe had to be added. Therefore, big toes of different sizes could be an explanation for the variability in foot length mentioned by Gay Robins.³⁷

Points D and E lack horizontal alignment with a vertical grid line. Nevertheless, the angles δ and ϵ are constant within narrow limits (see Tab. 2). The constancy of the length ratios d/b (1.51 ± 0.05 h, RSD 3.4%) and e/a (1.55 ± 0.05 h, RSD 3.3%) is also striking.³⁸ This supports hypothesis 3 that points D and E on H14 were marked with a compass. The circle radii (d , e) could serve as guide for the anterior and posterior chest contours (hypothesis 3).

As an objection to H2 and H3, it can be argued that points A, B, D and E were marked by plotting a canonical multiple of the module $\frac{1}{18}$ h on H0 and H14, respectively, on either side of the vertical midline (Fig. 5). Given the canonical specification of 0.5 h for the height of the leg triangle (c) and the hypothetical specification of $\sim 28^\circ$ for the angle γ , the step width $A \rightarrow B$ is 0.25 h ($\frac{4.5}{18}$ h). The mean step width measured in this study is actually 0.25 ± 0.02 h (RSD 7.1%). So the instruction to the artists might have been to plot $\frac{2.25}{18}$ on the base line on either side of the vertical midline in order to achieve the correct heel positions. As for points D and E on H14, the distance $m \rightarrow D$ in this study is 0.08 ± 0.02 h (RSD 24.8%) and the distance $m \rightarrow E$ is 0.12 ± 0.02 h (RSD 14.4%). So the instruction might have been to plot $\frac{1.5}{18}$ h for the distance $m \rightarrow D$ and $\frac{2}{18}$ h for the distance $m \rightarrow E$. However, given the two-digit RSDs, the question arises as to why these targets were

³⁴ This retraction is usually more pronounced on the rear foot than the front foot.

³⁵ Cf. Robins 1994, 36 (fig. 2.5), 49 (fig. 2.10).

³⁶ Cf. Darlow 2017, 197, 215 (fig. 5.18).

³⁷ Robins found in his grid scheme that feet are about 3 squares long, sometimes longer and rarely shorter. Cf. Robins 1994, 74.

³⁸ The slightly longer line e ($e/d = 1.03 \pm 0.01$) was probably not intended by the artists.

missed so often at these two relatively short distances. In contrast, the length ratios d/b and e/a are much more precise with an RSD of 3.4% and 3.3%, respectively, even though lines d and e represent long diagonals. This is an argument for the correctness of hypothesis 3. The compass method also eliminates the question of how the angles γ and δ or the inclination of the anterior and posterior chest contours were obtained.

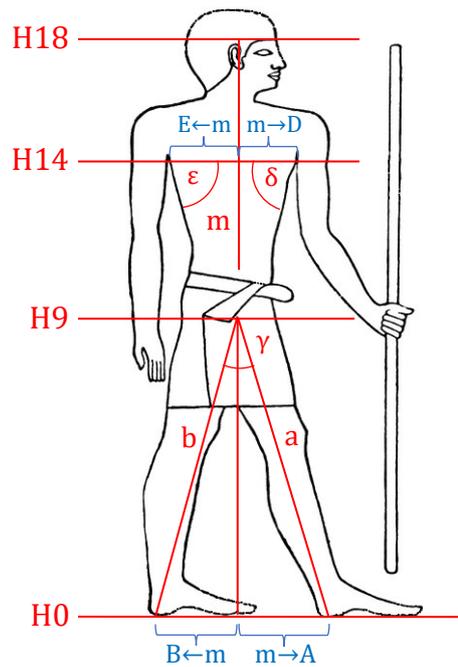


Fig. 5: Alternative construction of points A, B, C, and D.

The reason why B' instead of B is the lower end of d (Fig. 2) can only be speculated. It makes the angle δ larger than the angle ϵ ($\delta/\epsilon = 1.08 \pm 0.03$). This softening of the symmetry was probably intended to give the upper body a more natural shape.

Hypothesis 4 suggests that the shoulders and the waist region were shaped with only a rough orientation to H11, H14, H15, H16, D, and E, and that their horizontal extension has no fixed relationship to h . However, the values in Table 3 show that the shoulder widths with $0.33 \pm 0.01 h$ (RSD 4.3%) are on average $\frac{6}{18} h$, and the waist widths with $0.13 \pm 0.01 h$ (RSD 6.8%) are on average $\frac{2.3}{18} h$ (i.e. approximately two and a half grid spacings). These values are too "smooth" to ignore (also because of the single-digit RSDs). It should be remembered that the proposed system of horizontals and diagonals is ultimately based on the height c ($\frac{9}{18} h$). The construction lines d and e , which determine both the position

of points D and E and the width of the waist, are defined relative to the legs of the leg triangle (a, b), which in turn are a function of the angle γ and the height c. The already mentioned high variability of the distances $m \rightarrow D$ and $m \rightarrow E$ at height H14 must also be taken into account. (Fig. 5). Adding 1 grid space on each side for the upper arms should result in an equally variable shoulder width. Since this is not the case, there is still the possibility that the artists designed the shoulders and upper arms freely, but with the constraint of not exceeding a total shoulder width of 6 grid spaces.³⁹ Corrections to figure contours occasionally found in murals support the assumption that such semi-canonical standards existed.⁴⁰

The armpits appear to have been freely drawn. The tips of the armpits are not aligned with a horizontal guide line, but are between H14 and H15 (the rear armpit is usually slightly higher than the front armpit).⁴¹ This was probably intended by the artist to give a vivid impression of the figure.

The height of the navel also has no verifiable fixed relationship to the grid and construction lines. The claim that the waist width is always half the shoulder width⁴² could not be confirmed for the 24 Old Kingdom figures studied. The ratio of waist width to shoulder width (g/f) in this study is only 0.38 ± 0.02 (RSD 5.0%; Tab. 3).

When a belted apron is shown, the front end of the belt is positioned at or close to H10.⁴³ The inclination of the waistline seems to have been freely chosen by the artist. In this study of 18 figures with belts, the angle that the upper edge of the belt forms with the horizontal plane is 15.5 ± 4.4 degrees (RSD 32.6%!).

The construction principles presented here apply equally to all the striding figures analyzed, regardless of the substrate (tomb wall, stela, or pillar). Images of tomb owners, common men (#5, #9, #10, #13, #14, #18, #19), and male minors (#16, #22, #23) were included. It is known from careful observation of the finer details of the craftsmanship that the decoration of a tomb was done in teams and that the quality of the work varied within a tomb.⁴⁴ However, the present study found no evidence that the principles of

³⁹ In my opinion, the curvature of the buttocks and the contours of the arms and legs were also drawn free-hand. See also Darlow 2017, 170 and Pieke 2011, 222 (fig. 7), 226 (fig. 12). Campbell C. Edgar wrote in 1905 that the horizontals or grids served only as a rough guide. The artist "no doubt drew his figures with practiced ease and was content to come within reasonable closeness to the conventional standard" (quote from Robins 1994, 39).

⁴⁰ Cf. Robins 1994, 26.

⁴¹ Cf. Robins 1994, 36, 73 (fig. 4.8), 76.

⁴² Wall figures in the tomb of Ukhhotep I, 12th dynasty (Meir B2) and in the tomb of Ankhhor, Late Period (Theben-West TT414) are sometimes cited as evidence for this claim. Cf. Blackman 1915, plate 10; Bietak/Reiser-Haslauer 1982, 226–227 and fig. 109.

⁴³ See also Robins 1994, 74.

⁴⁴ Cf. Robins 1994, 26; Pieke 2011, 216–217, 222–225.

construction were applied differently or more sloppily to “less important” figures than to the elite tomb owners. Similarly, no anomalies were found in the case of minors compared to adults.⁴⁵

4.2 Possible practical approaches

Two mathematical papyri, the Moscow Mathematical Papyrus from the 11th dynasty⁴⁶ and the Rhind papyrus from the early Second Intermediate Period,⁴⁷ prove what one might have guessed from the grandiose architecture: advanced arithmetic and geometry were practiced in ancient Egypt. For example, the Rhind papyrus contains methods for calculating the area, volume, and inclination of a pyramid as well as instructions for calculating the area of a circle. An approximate value for the number π (3.16049) was given in the form of an addition of fractions.⁴⁸ In general, fractions were an essential feature of ancient Egyptian mathematics.⁴⁹ Artists had various measures of length and more or less precisely calibrated measuring rods and measuring cords at their disposal since the early Old Kingdom.⁵⁰ There is no direct evidence that they had instruments for measuring angles, but they did know certain laws of trigonometry.⁵¹

The procedures for the drawing of guide lines or grids can be derived from techniques still used by painters today. You can rub paint on a piece of string, stretch it between the desired end points of a straight line and snap it lightly against the ground.⁵² To construct a right angle, you can use a simple trigonometric law, often called the “3-4-5 rule”.⁵³

A simple compass (string compass) consists of a string as long as the desired radius, with a sharpened piece of charcoal, ochre, or raddle at one end, and the other end fixed at the intended center of the circle. Greek geometry in the first millennium BC was a

⁴⁵ Ancient Egyptian art ignored the specific body proportions of children.

⁴⁶ Also known as the Goleniščev papyrus (Pushkin State Museum of Fine Arts, Moscow).

⁴⁷ British Museum, EA10057 and EA10058 (see the museum website). The papyrus is a copy, the original probably dates from the 12th dynasty.

⁴⁸ Cf. Clagett 1999, 75–93; Eisenlohr 1891, 75–132.

⁴⁹ Cf. Clagett 1999, 20–42; Eisenlohr 1891, *passim*.

⁵⁰ Cf. Clagett 1999, 7–11; Eisenlohr 1891, 8–10; Minow 1992, 2–5.

⁵¹ For example, in the papyrus Rhind, tasks 56–60. Cf. Eisenlohr 1891, 134–149; Clagett 1999, 70–73.

⁵² Imprints of loose fibers of the cord and small splashes of paint caused by this technique can sometimes be seen on unfinished ancient Egyptian wall paintings. Cf. Robins 1994, 26. The word “line” is etymologically derived from the Latin word *linea*, meaning “line, cord, thread” (originally the taut cord used to draw a line, later also the line itself). Cf. DWDS website, search term “*Linie*”.

⁵³ In a triangle with a side length ratio of 3:4:5, the sides with relative lengths of 3 and 4 form a right angle. This technique is the basis for the 12-knot cord and the ceremony of “tightening the ropes” before constructing a building. The method was also known in ancient India and China. Cf. Hankel 1874, 83; Minow 1992, 2–8, 11 and fig. 5. 12.

development of Egyptian geometry.⁵⁴ The Greek scholar Plato (427–347 BC) called for the maintenance of "purity" in geometry, where no tools other than a ruler and a compass were permitted.⁵⁵

The formula given in Chapter 1.2 for calculating the radius r of the arc with center C requires the cosine of $\gamma/2$ in radians (rad). It is possible that the ancient Egyptian artists used radians for angles (full angle = $2\pi \approx 6.28$ rad) and knew the cosine.⁵⁶ But it is more likely that they knew from a long tradition that in order to determine the compass radius to mark the positions of A and B on the base line, they had to extend c beyond the base line by an amount u . Substituting an angle γ of 28° (0.489 rad) into the formula on page 7, we obtain $u = 0.0306c \approx 0.031c$ and $r = a = b \approx c + 0.031c$.⁵⁷ This value for u could be approximated by dividing one grid spacing into 4 equal parts (1 part = $u = \frac{1}{4} \times \frac{1}{9}c \approx 0.028c$) or by dividing 3 grid spacings into 10 equal parts (1 part = $u = \frac{1}{10} \times \frac{1}{3}c \approx 0.033c$).⁵⁸ The geometric division of a line into equal parts can be done very easily with a compass based on the intercept theorem.⁵⁹

According to hypothesis 3, length relations were decisive for the construction of points D and E . Specific target ratios d/b and e/a (1.51 ± 0.05 and 1.55 ± 0.05 , respectively, in this study) were used to calculate the radius of the arc with center B' used to mark D on H14, and the radius of the arc with center A used to mark E on H14 (Fig. 2). It seems that the radius previously used to mark A and B on H0 was simply increased by one and a half times.

The methods proposed here are simple and time-saving and fit in well with the well-known preference of the ancient Egyptians for calculating with fractions, even and especially in geometry.⁶⁰ Marking the points A , B , D and E with a compass the cord of which is rubbed with paint would have the advantage that the legs of the leg triangle (a ,

⁵⁴ Cf. Hankel 1874, 1–2, 73–76, 77–78, 83–89.

⁵⁵ Cf. Hankel 1874, 155–156.

⁵⁶ This cannot be ruled out since the stride angle is divided by the vertical midline into two right-angled triangles, and the cosine is a simple ratio (adjacent/hypotenuse).

⁵⁷ The canonical angle γ of 28° was found to be plausible in 24 figures (Tab. 2). The range of 24.6 – 31.5° in this sample can be explained by the fact that a slight inaccuracy in determining the distance u leads to a relatively large change in the angle γ . This was certainly a challenge with smaller figures (servants, offering bearers).

⁵⁸ It is known that the distance between two horizontals was divided into further parts if necessary. Cf. Darlow 2017, 183–184; Robins 1994, 71 (fig. 4.6), 72 (fig. 4.7), 82 (fig. 4.16). The exact mean value for the angle γ in this study ($n=24$), namely 28.1° (0.49 rad), would be obtained with $u = \frac{5}{18} \times \frac{1}{9}c \approx 0.0308c$. These are only examples, the exact way how the artists determined the distance u should not be postulated here.

⁵⁹ It seems possible that the proportion grid was also constructed with a compass.

⁶⁰ Cf. Eisenlohr 1891, 118–149.

b) and the construction lines of the chest (d, e), could be marked by snapping the compass cord against the wall. This is especially useful for large figures. However, traces of the hypothetical lines have not yet been described for wall paintings. Within the legs and chest, they disappeared at the latest when these areas were painted reddish brown⁶¹ or when the relief was hammered.⁶² The observation that a grid was not essential in the Old Kingdom⁶³ could be taken as further evidence for the use of a compass.

4.3 Limitations of the study

This study has the character of a pilot study, limited for the time being to Old Kingdom reliefs. The sample size of 24 is relatively small, but allows for valid statistical analysis.

The analysis was performed on photos, as it was not possible to work on the originals. Some perspective distortion is to be expected in photos, especially if the angle of view is tilted upward for large-format motifs or motifs above eye level.⁶⁴ On the other hand, this is also the perspective of the tomb visitors.⁶⁵ In any case, the relatively small range of the values collected shows that this factor did not reduce the validity of the study.

4.4 Closing remarks

The hypotheses raised in this article are plausible, but, of course, cannot be confirmed. In conclusion, the heights of certain anatomical landmarks were standardized in relation to the height of the frontal hairline, but the grid (if present) was basically "only" to facilitate the transfer of a sketch to the larger scale of the wall.⁶⁶ This was a great advantage in complex scenes with different postures of the figures involved or different scaling within a scene.⁶⁷

The construction principles of striding figures proposed in this work are based on horizontals and angles constructed with the help of two circle radii. These two radii are in a defined length ratio to each other. This is reminiscent of the geometric morphometry

⁶¹ The reddish skin color is one of the reasons why guide lines and preliminary contours were generally drawn in red. Cf. Robins 1994, 26; Pieke 2011, 227.

⁶² Traces of guide lines are rarely found within the outlines of figures. Cf. Darlow 2017, 207 (fig. 5.4), 208 (fig. 5.6), 209 (fig. 5.7). Wall paintings are an exception. Cf. Darlow 2017, 212 (fig. 5.13), 214 (fig. 5.17).

⁶³ Cf. Baud 1935, 44–58; Blackman 1915, Plates 10, 11,15; Robins 1994, 60, 64, 66.

⁶⁴ This problem was also addressed by Robins, who worked with grids. He therefore photographed wall decorations with a tilt-and-shift lens that could be extended upwards. Cf. Robins 1994, 62–63. I do not know whether the photos used for this study were taken in this way.

⁶⁵ The artists of antiquity took this into account. When only the heads of monumental statues are exhibited in museums, their faces sometimes appear grotesquely distorted because they were made for a different perspective than that of the museum visitor. See also Pieke 2011, 225–227.

⁶⁶ Cf. Robins 1994, 31, 37–38.

⁶⁷ Examples in Robins 1994, 186 (fig. 8.2), 188 (fig. 8.3), 193 (fig. 8.6), 194 (fig. 8.7).

used in modern comparative anthropology. There, a distinction is made between the shape and the form of an object being studied (e.g. a body part or skeletal component). Unlike form, shape is independent of the size of the object. Shape variables include point coordinates in a Cartesian coordinate system (two- or three-dimensional), angles, and distance ratios. They allow the object to be moved, rotated and scaled to any size within the coordinate system without changing its shape.⁶⁸

A larger study that will include reliefs from later periods is in preparation. In addition to the angles γ , δ and ϵ , the arm and knee angles of seated and kneeling figures also give the impression of being canonical and should be checked accordingly.

⁶⁸ An overview is given in Mitteroecker/Gunz 2009, 235–247.

Abbreviations

ASE: Archaeological Survey of Egypt; DGOeAW: Denkschriften der Gesamtkademie, Österreichische Akademie der Wissenschaften (Vienna); DWDS: Digitales Wörterbuch der Deutschen Sprache (Berlin-Brandenburg Academy of Sciences, Berlin); Dyn.: dynasty; H: horizontal line; JPEG: Joint Photographic Experts Group (format); Mem. Phil.: Memoirs of the American Philosophical Society Held at Philadelphia for Promoting Useful Knowledge (Philadelphia); NA: not assessable; RSD: relative standard deviation; RT: Recueil de travaux relatifs à la philologie et à l'archéologie égyptiennes et assyriennes; SD: standard deviation; TT: Theban tomb.

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